EFFECTS OF COLLABORATIVE REVISION ON UNDERGRADUATE STUDENTS' PROOF VALIDATION SKILLS

Emily Cilli-Turner
University of Illinois at Chicago
ecilli2@uic.edu

Although the ability to determine the validity of a proof is an important skill for mathematicians and mathematics educators alike, there is ample evidence that undergraduate students in a transition to proof course are not proficient at this task. This study employs a teaching intervention called collaborative revision, which refers to the process in which students present a proof they have constructed to their classmates who are encouraged to provide feedback to aid in the revision of the proof. Pre- and post-assessments were administered to students and interviews were conducted to measure students' proof validation skills and to investigate if collaborative revision impacts these skills. Results show that although collaborative revision may not impact students' abilities to identify valid proofs, it does affect the way that students gain conviction about the proof of a statement.

Keywords: Reasoning and Proof, Classroom Discourse

Introduction

This study aims to investigate the impact on argument validation skills of students after participating in a process called collaborative revision in an introduction to proof course. *Collaborative revision* refers to the process in which students present a proof they have written to their classmates and the other students are encouraged to make comments and point out inconsistencies in order to ensure that the proof is valid. Based on feedback from classmates, the student then revises the proof and presents it again, repeating the process until the proof is valid and includes all the relevant details. In this context, this research aims to answer the following research question: To what extent does participation in collaborative revision impact undergraduate students' proof validation skills?

Related Literature

Rav (1999) proposes and Hanna & Barbeau (2009) agree that proofs are of the utmost importance in mathematics, since they, instead of theorems, are the main vehicles in which mathematical knowledge is contained and transferred. Thus, the ability to determine if a given proof is valid becomes an important skill for students. Selden & Selden (2003) highlight that this skill is invaluable for not only future mathematics educators, since they will someday have to evaluate student proofs for assessment purposes, but also for future mathematicians because they will have to examine proofs to learn about new mathematics being produced. Additionally, proof validation is intricately linked to proof construction so students in a transition to proof course need both sets of skills (Selden & Selden, 2003).

Selden and Selden (2003) found that undergraduate students have trouble differentiating between a valid and an invalid proof. In a proof verification task, undergraduate students in a transition to proof course initially judged proofs correctly less than half the time. However, when prompted for reflection about the proofs by the interviewer, the students were able to correctly evaluate proofs at over 80%. The authors conjecture that this may be due to the fact that proof verification skills are not explicitly taught to students and the texts for these courses

include very few proof validation tasks, with those that do exist usually having just a single error to be detected. Alcock & Weber (2005) found similar results with students in an undergraduate real analysis course and write that "this suggests that the ability to validate proofs may be in many students' zone of proximal development and that students' abilities in this regard might improve substantially with relatively little instruction" (pg. 131). Thus, this study aims to determine the effect that making proof validation a part of normal classroom activities will have on students' ability to correctly judge the validity of proofs.

There is much research to support the hypothesis that collaborative learning can greatly enhance student learning about mathematical proof. Yackel & Cobb (1996) note that participation in a community of learners can be a vital part of students' success in mathematics. Additionally, a study by Strickland & Rand (2012) allowed students to submit multiple revisions of proofs in response to teacher feedback and measured the effects on student learning. The teacher comments given were minimal, often just circling a confusing or incorrect passage of the proof, and students were allowed as many revisions as needed. Although the data set was small, on average, students in the revision group did better on the final exam. Proof validation studies (Selden & Selden, 2003; Alcock & Weber, 2005) also show that reflection about written proofs can aid in correctly evaluating the validity of a given proof. Thus, collaborative revision is a way to explore the benefits of combining these proven techniques and this study examines the impact on students' validation skills when using a collaborative revision teaching intervention in an introduction to proof classroom.

Methodology

Context & Participants

This study was enacted at large Midwestern University in courses offered by the Mathematics, Statistics & Computer Science department. The collaborative revision teaching experiment took place in a course correlated with a transition to proof course offered through the Emerging Scholars Program at the university. Students were able to self-enroll in this treatment course and, thus, this is a quasi-experimental study. Since the treatment course was supplemental, the majority of students in the treatment course were concurrently enrolled in the correlated lecture-based transition to proof course, which is the comparison course in this study.

Treatment group participants were drawn from the treatment course in this study and there were 15 students in this group. A comparison group was desired to determine the impacts of the teaching experiment when compared to a lecture-based course. The comparison group solicited students from the comparison course and was comprised of 12 students. Additionally, two students from the comparison group and two students from the treatment group were interviewed to gain insight about what aspects of a proof are convincing to students in each group.

Course Design

Each week students in the treatment course were expected to come to class with a written proof of a statement given by the instructor. At the beginning of class, students were put into small groups (no more than 4 per group) and they discussed the proofs one at a time. During this time, the students were encouraged to carefully read their classmate's proof, writing down things as necessary, and verbalize their impressions of the proofs. Based on the feedback given, the student was responsible for revising the proof and bringing it back in the following week. The treatment course often exposed students to proofs from their classmates, which were neither valid nor written in the most rigorous way. In the context of determining the effects of collaborative revision on student proof validation skills, it was hypothesized that students in this

experimental course would be more proficient at identifying invalid proofs since they would have explicit practice doing this during the course of the semester.

Data Sources

To measure the effects of the collaborative revision on proof validation, a pre and post-assessment was given to all participants. The assessment required students to examine four 'proofs' of a given statement and determine whether each was a valid or invalid proof. The arguments presented to the students were adapted for this study from the proofs given to high school students in Healy and Hoyles (2000) to be appropriate for undergraduates. On the assessment, students were first asked to determine if each proof was valid or invalid, with these terms intentionally undefined to see if students would gain more of an understanding of what a valid proof entails throughout the course of the semester. There were also two more questions on a three-point Likert scale asking students how well they felt they understood each argument and how certain they were about their classification. The same proofs to be validated were given on the pre and post-assessment.

In addition to the assessments, student interviews were conducted with two students in the treatment course and two students in the comparison course. Students were interviewed individually twice during the course of the semester; first shortly after the pre-assessment was administered and again shortly after the post-assessment was administered. The questions asked to students, according to Zazkis and Hazzan (1998), are performance questions, unexpected "why" questions and reflection questions, requiring students to explain why they chose each argument as valid or invalid. During the second interview, students were shown their answers on the pre-assessment and asked to compare to the post-assessment and explain any inconsistencies. This was done to determine what aspects of an argument convinces students, called *student proof schemes* by Harel & Sowder (1998), and if this evolved during this semester.

Data Analysis & Results

Data from the assessments (pre and post) was analyzed to determine the percentage of correct classifications made by each student. Additionally, descriptive statistics were computed for the Likert ratings regarding students' self-reported understanding and certainty about classification for each of the four arguments and independent samples t-tests were done to determine differences between groups (i.e. treatment and comparison). Student interviews were transcribed and analyzed to determine the proof schemes exhibited by each student during the pre and post-interviews.

Consistent with prior proof validation studies (Alcock & Weber, 2005; Selden & Selden, 2003), this study found that students very often incorrectly classified proofs. On the pre-assessment, students in the treatment course were able to classify proofs correctly at an average rate of 60%, while students in the comparison course had a rate of 45%. However, a t-test showed this difference was non-significant ($p = 0.2$). On the post-assessment, the comparison group increased their average correct to 54%, while the treatment group's average remained unchanged. This suggests that the collaborative revision process had little impact on students' proof validation skills. Even though the treatment group did not show an increase in average percentage of proofs correctly classified from the pre to the post-assessment, they did report a significantly better understanding than the comparison group on almost every proof. Additionally, the treatment group reported significantly higher confidence of their classification than the comparison group on two out of four proofs. So, collaborative revision has an impact on students' ability to understand a written proof and their certainty about classifying that proof.

Results from the interviews show that the students interviewed from the treatment group exhibited higher-level proof schemes of Harel & Sowder (1998) from the pre-assessment to the post-assessment, while the proof schemes of students interviewed from the comparison group remained largely unchanged. Table 1 shows the proof schemes that each student interviewed (pseudonyms are used) exhibited during each of the interviews. These results imply that even though collaborative revision has little impact on students' abilities to identify valid proofs, it does impact the proof schemes held by students and how they gain conviction about the proof of a statement.

Table 1: Proof Schemes Exhibited by Students in Each Group During Each Interview

Group	Student	First Interview	Second Interview
Treatment	Stephanie	Ritual	Transformational
	Robert	Internalized	Intuitive-Axiomatic
Comparison	James	Empirical	Transformational
	Francine	Transformational	Transformational

Acknowledgment

The author acknowledges Dr. Mara Martinez for her supervision and helpful comments on this research project.

References

Alcock, L. & Weber, K. (2005). Proof validation in real analysis: Inferring and checking warrants. *Journal of mathematical behavior, 24*, 125-134.

Hanna, G., & Barbeau, E. (2009). Proofs as bearers of mathematical knowledge. In G. Hanna, H. Jahnke, & H. Pulte (Eds.), *Explanation and proof in mathematics: Philosophical and educational perspectives* (pp. 85–100). New York, NY: Springer.

Harel, G. and L. Sowder (1998). Students' proof schemes: Results from exploratory studies. *CBMS issues in mathematics education* 7, 234–283.

Healy, L. & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for research in mathematics education, 31*(4), 396–428.

Rav, Y. (1999). Why do we prove theorems? *Philosophia mathematica*, *7*(1), 5–41.

Selden, A. & Selden, J. (2003). Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem? *Journal for research in mathematics education, 34*(1), 4–36.

Strickland, S. & Rand, B. (2012). Improving undergraduates' novice proof-writing: Investigating the use of multiple drafts. In S. Brown, S. Larsen, K. Marrongelle, and M. Oehrtman (Eds.), *Proceedings of the 15th Annual Conference on Research in Undergraduate Mathematics Education*, Portland, Oregon, pp. 555–559.

Weber, K. (2002). Beyond proving and explaining: Proofs that justify the use of definitions and axiomatic structures and proofs that illustrate technique. *For the learning of mathematics, 22*(3), 14–17.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for research in mathematics education, 27*(4), 458–477.

Zazkis, R. and O. Hazzan (1998). Interviewing in mathematics education research: Choosing the questions. *Journal of mathematical behavior, 17*(4), 429–439.