ARGUMENTATION IN UNDERGRADUATE MATH COURSES: A STUDY ON DEFINITION CONSTRUCTION

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The purpose of this study is to analyze the complex argumentative structure in undergraduate mathematics classroom conversations during definition construction by taking into consideration students' and teacher' utterances in the classroom using field-independent Toulmin's theory of argumentation . The analyses contributed to an emerging body of research on classroom conversations.

INTRODUCTION

Definitions express the properties that characterize the 'objects' of the theory and relate them within a net of stated relations (Mariotti & Fischbein, 1997). In constructing definitions arguments must be brought to support the thought processes in constructing them. In mathematics education literature the argument concept is used in the sense of justifying a conclusion based on a data (Toulmin 2003; Mejia-Ramos & Inglis, 2009; Knipping, 2008). On the other hand, argumentation is a verbal and social activity of reason aimed at increasing (or decreasing) the acceptability of a controversial standpoint for the listener or reader, by putting forward a constellation of propositions intended to justify (or refute) the standpoint before a rational judge' (van Eemeren et al, 1996).

As Toulmin's model is intended to be applicable to arguments in any field, it has provided researchers in mathematics education with a useful tool for research, including formal and informal arguments in classrooms (Knipping, 2008). Studies using Toulmin model focused on analyzing students' arguments and argumentations in proving processes in a classroom (Knipping, 2002, 2008; Krummheuer, 1995) and, individual students' arguments in proving processes (Pedemonte, 2007). Toulmin himself noted that his ideas has no finality. Indeed his model has been reshaped in various ways, his claims have been contested by some and in response reformulated by others, and some but not all aspects of his approach have been incorporated in applications in different domains (Hitchcock & Verheij, 2006).

Having established these facts, the goal of our research is to study the argumentation in undergraduate mathematics classrooms during definition construction using Toulmin's theory of argumentation. Specifically, the aim is to analyze the structure of the arguments accomplished in the course of interaction where the teacher and students involvement in this accomplishment. This study is part of a wider study investigating the argumentation generated in undergraduate mathematics classes while proof generation (see Ubuz, et al, 2012), definition construction, and problem solving. Here we concentrate only definition construction because of page restrictions. This paper suggests a method by which complex argumentation in defining processes can be reconstructed and analyzed. Analyzing students' and teacher' utterances in the

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classroom according to Toulmin model allows us to reconstruct argumentations evolving in the classroom talk since arguments are produced by several students together with the guidance of the teacher.

THEORETICAL FRAMEWORK

In the following sections we will expose some theoretical considerations on the Toulmin model, and the definition construction process.

The Toulmin Model

According to Toulmin, an argument is like an organism. It has both a gross, anatomical structure and a finer, as-it-were physiological one (Toulmin, 2003). He is interested in the finer structure. The Toulmin model is differed from analysis of Arisitotle's logic from premises to conclusion. First, we make a claim(C) by asserting something. For the challenger who asks "What have you got to go on ?", the facts we appeal to as foundation for our claim is called data (D) by Toulmin. After producing our data, we may being asked another question like "How do you get there ?". He notes, at this point we have to show that the step from our data to our conclusion is appropriate one by giving different kind of propositions like rules, principals, inference – licenses or what you will, instead of additional items of information (Toulmin, 2003). A proposition of this form Toulmin calls a warrant (W). He notes that warrants are of different kinds and may confer different degrees of force on the conclusions they justify. We may have to put in a qualifier (Q) such as "necessarily", "probably" or "presumably" to the degree of force which our data confer on our claim in virtue of our warrant. However there may be cases such that the exceptional conditions which might be capable of defeating or rebutting the warranted conclusion. These exceptional conditions Toulmin calls as rebuttal (R). For our challenger may question the general acceptability of our warrant: "Why do you think that?" Toulmin calls our answer to this question our backing (B) (Hitchcock & Verheij, 2006). The diagram of the Toulmin model is as follows :

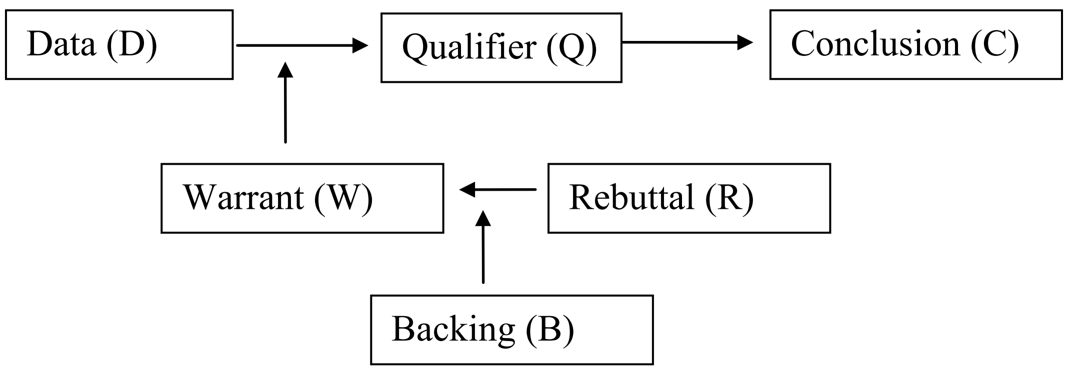

Figure 1: The Toulmin Model

Reconstructing and analyzing the complex argumentative structure in classroom conversations follow their own structure. For example, careful analyses of the types of warrants (and backings) that students and teachers employ in classroom situations allowed two distinctions in the justifications: visual and conceptual (Knipping, 2008). The warrants and backings based on conceptual aspect or deductive are mathematical concepts or mathematical relations between concepts, and make reference to theorems, definitions, axioms and rules of logic. The warrants and backings based on visual or figural aspect make reference to figures as part of the argumentation.

Definition Construction Process

Defining is a basic component of mathematical knowledge, and learning to define is a basic problem of mathematical education (Mariotti and Fischbein, 1997). Ouvrier – Buffet (2006) reported that Situation(s) of Definition Construction gave the opportunity to work on scientific processes (construction of definitions and proof in particular). "Scientific processes are constituted by students' experiments with different cognitive attitudes: doubting, conjecturing, refuting, generating new counter-examples, testing etc" (Ouvrier – Buffet, 2006; p.279). Ouvrier – Buffet (2006) and Mariotti and Fischbein (1997) have pointed out that the intervention of the teacher plays a central role in guiding the discussion and mediating the definition construction process. In addition, Ouvrier – Buffet (2006) have emphasised that active connections have to be underlined in defining processes.

METHODOLOGY

Data were collected through nonparticipant observations that were videotaped. Observation was conducted 2009-2010 spring semesters in real analysis course for eight weeks, and 2010-2011 spring semesters in advanced calculus course for six weeks, offered to mathematics education student at the third and second years, respectively. These courses were selected as both formal and informal argumentations were at the focus of these courses. In these courses the number of students were 45 and 40, respectively. Formal proof approaches are given to the students at the "Abstract Mathematics I - II" courses provided in the first year. In these courses, students learn what a proof is and how to prove theorems. That is, they learn how to argue mathematically, justify their claims and encounter the cases named "counter example" for the first time which rebuttals their claims.

The analysis of the observations is based on the transcripts. As Toulmin(2003) noted, "an argument is like an organism. When set out explicitly in all its detail, it may occupy a number of printed pages or take perhaps a quarter of an hour to deliver; and within this time or space one can distinguish the main phases marking the progress of the argument from the initial statement of an unsettled problem to the final presentation of a conclusion" (p. 87). Based on this explanation, eleven argumentations were determined and two of them were on definition construction. These two argumentations were observed in real analysis course.

Observations were conducted by the second author. He analyzed the transcripts by marking the progress of the argument from the initial statement to the final conclusion through using Toulmin model components. He noticed that some aspects of observed argumentations were overlooked. He modified the Toulmin model by integrating guide – backing and guide – redirecting additional components which were observed in almost all argumentations. We called an approval given by teacher to the warrants, backings or intermediate conclusion as guide – backing. When the argumentation does not start from a right point or students get stuck on an argument point, teacher intervenes with an example, a question or a suggestion to arrange the argument. We called such intervenes as guide – redirecting.

Having discussed with the first author who is a full professor in mathematics education and doing research on proof, it was decided that observed argumentations could be considered into three classes: proof generation, definition construction, and problem solving. She also noted that some components could be classified in itself. After re-analyzing observed argumentations, *warrant* component were divided in two categories: *deductive warrant* and *reference warrant*. Students appeal reasoning like numerical computing, applying a rule to an inequality, creating new ideas from a definition, a theorem or a rule in producing their warrants. We called this kind of warrants as *deductive warrants* as Inglis et al. (2007) did. When a warrant referred to a theorem, a definition, a rule or a problem, we called such a warrant as *reference warrant*. *Guide – backing* was divided into three categories: *approval*, *reference* and *terminator*. When teacher just approve the students' warrant, backing or conclusion by saying "good, fine, great, well done" and does not use any mathematical phrase, we called this kind of guide backing as *approval guide backing*. When teacher approve the students' warrant, backing or conclusion by referring a definition, a theorem or a problem recently solved, we called this kind of guide backing as *reference guide backing*. Argumentations come to an end when teacher or students reach the final conclusion to be achieved. In case, teacher reaches the final conclusion, students convince that the conclusion is legitimate. In case, students reach the final conclusion, teacher serves a backing. This backing shows the final conclusion and we called it as *terminator guide backing*. One important point that must be noted here is that argumentations were not analyzed according to their mathematically correctness.

Finally, full transcriptions together with analysis model components explanation are provided to an external auditor who is a researcher in mathematics education field. After a week, the auditor completed her analysis and a complete consensus was reached on analysis of argumentations.

RESULT

Two definition construction cases constituted two different argumentation context. In this paper only one case is considered as an example because of page restrictions. Here we analyze a transcript of an argumentation in which deductive warrant, guide – redirecting, approval guide – backing and terminator guide – backing appear. The following argumentation occurred when defining distance from a point to a set.

1 Teac : Well…Our goal is how to define the distance from a point to a set . At first let's say our work is on R. Let A be a set on R, say A = [-2, 2) and P be a point in the exterior of A. Well…Fatma, what is your comment on the distance from P to A.

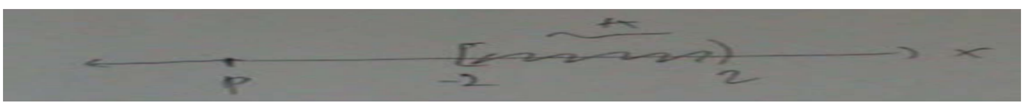

Figure 2 : The point P and the set [-2, 2)

2 Fatma : I think, one is from P to -2 and the other one is from P to 2.

3 Teac : Yes

4 Fatma : Umm…I am thinking…The distance we are looking for should be between these two values but not predicting its exact value.

5 Teac :Are you saying “I am not predicting it” or something symbolic ?

6 Fatma : Well, I am taking the absolute value of 2 - P and - 2-P .

7 Teac : You are focusing on |2 - P| and | - 2 - P|.

8 Fatma : I think that the distance we are looking for should be one of them.

9 Teac : Which one then ?

10 Fatma : Umm…I think that the distance we are looking for should be between them.

11 Teac : Your thoughts are essentially true. Well, you calculate | 2 - P| and |-2 - P| and according to you one of them is the distance we are seeking. So, which one would you think?

12 Fatma : It could be the small one or the big one or between them since A has elements between -2 and 2.

13 Teac : Let’s take an example from real life. Suppose that we are travelling to Istanbul. Seeing the city boarder on our way then you would say that “we are in Istanbul”, right?

14 Class : The closest one.

15 Teac : The closest one ? What does it mean in mathematics ?

16 Ahmet : The smallest one.

17 Teac : Fine, nice. Well, what if our set A is in that form (see Figure 3)

Figure 3 : The point P and another set

18 Zeynep : Calculate distance from P to every point of A which means taking the absolute values. Then the minimum of these values is the distance from P to A.

19 Teac : Well done! That’s it! Her thought is correct.

In line 2, Fatma considered the distances from P to 2 and from P to -2 as her data. Based on these data, in line 4 she concluded that the desired distance from P to A must be between these distances. The teacher, in line 5, gave a guide – redirecting by expecting a symbolic statement. Thereupon Fatma modified her data by using absolute value concept and pointed out in line 8 that the desired distance must be one of the absolute values, but in line 10 she returned to her old conclusion. In line 13, the teacher gave first but weak approval guide – backing for Fatma’s modified conclusion in line 11 and a guide – redirecting by asking her what it happen to be. In line 12, Fatma seems a bit confused. She combined her old conclusions in line 8 and 10 and produced a new conclusion. That A has elements between -2 and 2 and used it as a deductive warrant for this new conclusion. In line 13, teacher gave an example from real life as a guide – redirecting. Hereafter most of the students reached the closest point to P as conclusion in line 14. Teacher gave another guide – redirecting by asking the meaning of ‘the closest' in line 15. Ahmet, in line 16, responded it as the smallest absolute value which means his conclusion. In line 17, teacher gave an approval guide – backing by saying 'nice, fine' and a guide – redirecting by asking another form of A. In line 18, Zeynep gave the desired definition and got terminal guide – backing in line 19. The diagram corresponding to the argumentation above is provided in Figure 4.

CONCLUSIONS

The intervention of the teacher plays an important role in definition construction process as pointed out by Ouvrier – Buffet (2006) and Mariotti and Fischbein (1997). Construction of definitions and proof in particular is a basic component of mathematical knowledge. Teacher acts as a guide who exactly knows the path to follow i.e. where to start and to end the argumentation. During the argumentation if students follow the wrong path, get a false intermediate conclusion or get stuck in a point, teacher intervene the students to put them on the path in which they have to follow.

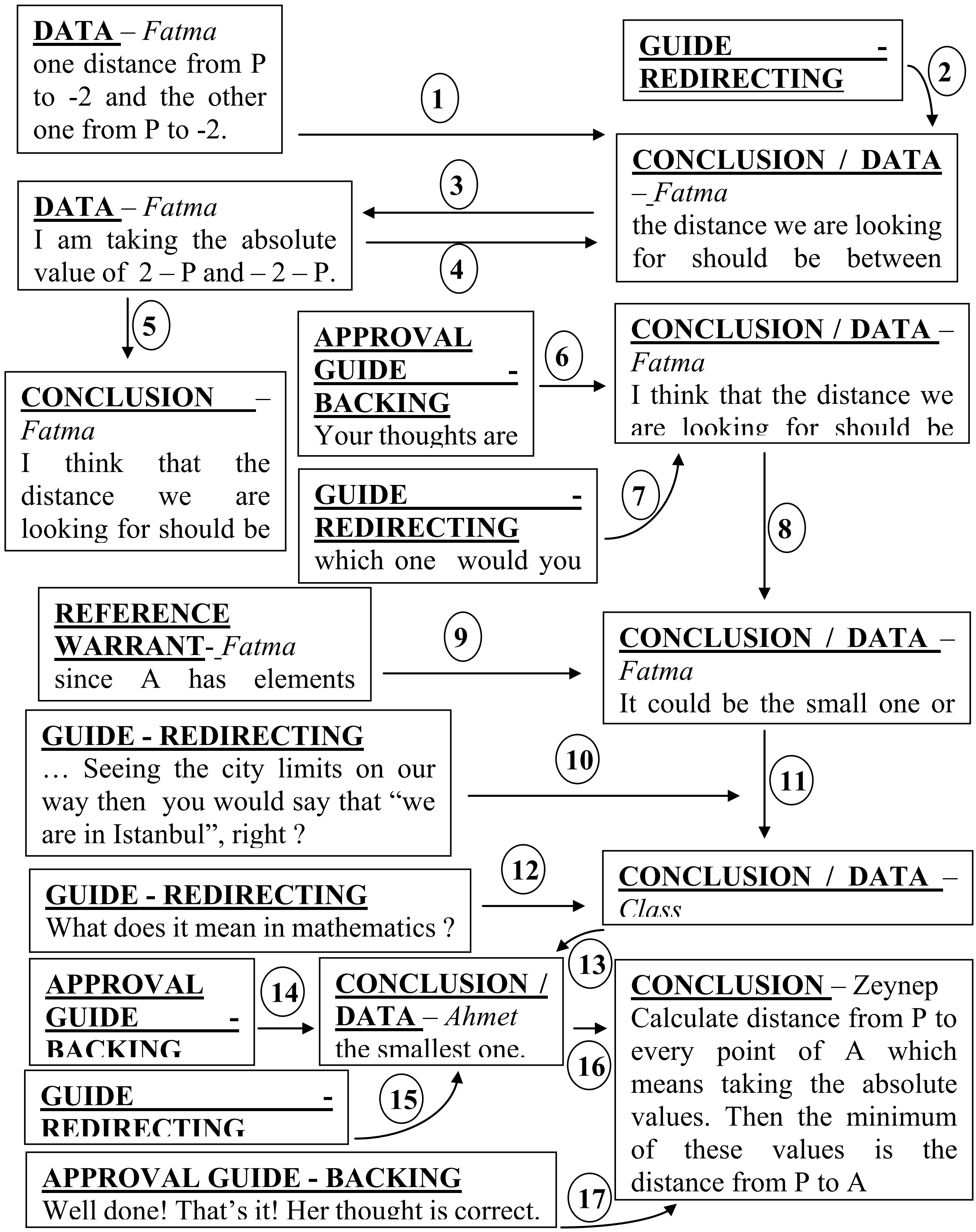

Figure 4: The Toulmin model of the argumentation

Based on our observations, the teacher played a role in argumentations by doing guide – backing and guide – redirecting. Mathematical process of defining incorporated two of the three categories of guide – backing component: approval and terminator. Careful analyses of the types of warrants that students and/or teachers employ in defining process allowed us to identify and classify only deductive warrant but not reference warrant. On the other hand, reference guide backing and reference warrant are encountered frequently in mathematical process of prove generation (Ubuz, et all, 2012).

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