FORMAL AND INFORMAL LANGUAGE IN MATHEMATICS CLASSROOM INTERACTION: A DIALOGIC PERSPECTIVE

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A perennial concern with the issue of informal and formal language in mathematics classrooms has led to an assumption that students must move from informal to formal mathematical expression as they learn mathematics. In this report, I draw on a Bakhtinian, dialogic perspective to examine formal and informal language in an elementary school mathematics classroom in Québec, Canada. The students, who are second language learners, are learning about polygons. I argue that informal and formal language are both necessary, and are always in tension.

INTRODUCTION

Many mathematics curricula now include an explicit focus on the development of mathematical language. Such development is generally conceptualised as a shift or transition from students' informal expressions of mathematical thinking, to communication using more standardised mathematical language. The Ontario elementary school mathematics curriculum, for example, suggests that in Grade 4, students should "communicate mathematical thinking […] using everyday language, a basic mathematical vocabulary, and a variety of representations" (OME, 2005, p. 65). By Grade 8, students are expected to use "mathematical vocabulary and a variety of appropriate representations, and observ[e] mathematical conventions" (p. 110). The use of 'everyday' language and 'basic' mathematical vocabulary has been replaced by 'mathematical vocabulary', implying a direction for development from the former to the latter. While this kind of approach seems reasonable, human language use is rarely so straightforward. In this report, I consider the nature of the relationship between informal and formal language in mathematics classroom interaction. To do so, I focus specifically on interaction in a classroom featuring second language learners of mathematics. The role of informal and formal language in such classrooms becomes particularly salient and significant.

FORMAL AND INFORMAL LANGUAGE IN MATHEMATICS CLASSROOMS

It is clear from the literature that the relationship between formal and informal language in mathematics is not entirely straightforward. For example, Pimm (1987) suggests that the appropriation of everyday language within the mathematics register may be a source of confusion for students. A word like 'difference', for example, has a common everyday meaning ('not the same') and a more specific mathematical meaning ('the result of a subtraction'). This observation suggests that there is a degree of overlap between informal, everyday language and mathematical language as it is commonly used. At the same time, research on mathematical discourse has increasingly highlighted the need to broaden the scope of inquiry to include a variety of

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meaning-making resources, including symbols, gestures and a variety of languages and language practices (e.g., Moschkovich, 2008). This work also shows that there are no clear-cut boundaries between mathematical language and everyday language; mathematical language is never entirely formal.

Research in classrooms featuring second language learners or bilingual or multilingual learners has also highlighted the issue of the development of formal mathematical language. In Adler's (2001) study in South Africa, for example, teachers reported a dilemma they experienced about whether to allow their multilingual students to express their mathematical ideas using informal language, or whether they should interrupt students' explanations to teach them formal, standardised mathematical language. This dilemma reflects a more widespread tension that has been observed in several other contexts. In multilingual classrooms, this tension interacts with tensions between students' home languages and the language of instruction and between language policy and classroom practice (see Barwell, 2012).

Both Setati and Adler (2001) and Clarkson (2009) have suggested that in multilingual or second language classrooms, students need to move along three different dimensions: informal to formal mathematical language; spoken to written mathematics; and home language to the language of instruction. This kind of approach is prevalent in the literature more generally and is apparent in the Ontario mathematics curriculum, as discussed above. This approach is productive and has generated valuable suggestions for classroom practice. It has not, however, interrogated sufficiently the nature of the *relationship* between informal, everyday expressions of mathematical thinking and more formal mathematical language. While there is recognition that the relationship is complex and that there are no clear boundaries, research is dominated by a model of transition from one to the other.

THEORETICAL FRAMEWORK: BAKHTIN'S DIALOGIC THEORY OF LANGUAGE

Although Bakhtin is primarily known as a literary theorist, his work includes a highly developed theory of language (see, in particular, Bakhtin, 1981). His theoretical perspective is wide-ranging and difficult to reduce to a list of simple tenets. For this report, I will summarise a few key ideas that are particularly relevant to the issue of formal vs. informal language. First, Bakhtin's theory of language is *dialogic*. This means that language use is dynamic and situated. In particular, any utterance is understood to be a response, one turn in an ever-unfolding chain of utterances, which "cannot fail to be oriented toward the 'already uttered,' the 'already known,' the 'common opinion' and so forth" (Bakhtin, 1981, p. 279). Moreover, each utterance is in dialogic relation with myriad alternatives, whether in terms of alternative pronunciation, choice of words, formulations, choice of language and so on. In this sense, dialogicality is a relational perspective on language. Meaning is made through the relations between sounds, words and utterances, not through these things 'in themselves'. Furthermore, the relationality of language is always towards what Bakhtin often calls 'an alien word', that is, difference or otherness.

Second, language precedes us. This means that we must always use the words of others, alien words, to express ourselves. But these words continue to express much that we may not intend:

> Language is not a neutral medium that passes freely and easily into the private property of the speaker's intentions; it is populated – overpopulated – with the intentions of others. Expropriating it, forcing it to submit to one's own intentions and accents, is a difficult and complicated process. (p. 294)

Hence any utterance is not simply an expression of an individual's idea; it expresses a host of 'other' ideas that derive from preceding usage and must be understood in the light of preceding utterances. Furthermore, since we can never escape from being in relation to the language that precedes us, this language in some sense defines who we are. For Wegerif (2008), "for each participant in a dialogue, the voice of the other is an outside perspective that contains them within it" (p. 353) (see also Radford, 2012).

Third, Bakhtin's theory of language includes a continual tension between a centripetal force towards uniformity ("unitary language") and a centrifugal force towards *heteroglossia,* which refers to the tremendous variety of language-in-use. This variety is related to social differences: "languages of social groups, 'professional' and 'generic' languages, languages of generations and so forth" (Bakhtin, 1981, p. 272), including the languages of mathematics, as well as the languages of social class, race, region and so on. The tension between informal and formal language, observed in many mathematics classrooms, is an instance of Bakhtin's more general tension. Formal mathematical language amounts to a unitary language, the idea of which exists in tension with the diverse forms of expression that students may use. Heteroglossia is an important aspect of dialogicality, since it is variation that leads to the continual interplay of different ideas, perspectives and meanings. As Holquist emphasises, however, "heteroglossia is a plurality of relations, not just a cacophony of different voices" (p. 89).

Finally, in the context of education, Wegerif (2008) contrasts a dialogic perspective with a dialectic perspective (which he associates with a neo-Vygotskian perspective on learning). From a dialectic perspective, differences must be overcome, synthesised into something new (and so, arguably, tending towards uniformity). It is, arguably, this perspective that informs the idea of a uni-directional process from informal to formal language. From a dialogic perspective, by contrast, differences open up possibilities for making meaning; the process is no longer necessarily uni-directional.

RESEARCH CONTEXT: LA CLASSE D'ACCUEIL

The work reported in this paper is from a project designed to examine mathematics learning in different second language settings in Canada, a country with two official languages, English and French. In this report, I refer to one of these settings: a sheltered class for new immigrant learners of French, known as a *classe d'accueil*. In the province of Québec, new immigrant children must attend school in French. If they do not speak French, they attend a *classe d'accueil* for up to a year to learn enough French to join mainstream classes.

I visited a Grade 5-6 class (10-12 years), along with a research assistant, towards the end of the school year, by which time the students had acquired a degree of basic French. Between us, we attended all the students' mathematics lessons for a period of three weeks. We made audio-recordings of whole-class interaction, as well as some small-group interaction. We also collected examples of students' written work and interviewed small groups of students and the teacher. The class comprised 18 students with a variety of origins and language backgrounds, including South Asian, West African and South American. The teacher reported that the main aim of the class was to prepare the students for school life in Québec and to learn to speak and think in French. In mathematics, she focused on vocabulary. All mathematics texts used in class were in French and the teacher insisted on the use of French at all times.

The analysis reported below examines the first of a sequence of three lessons introducing some language and concepts in geometry. During these lessons, the main emphasis was on learning the concepts and words for polygon, non-polygon, convex and non-convex. For this report, I focus particularly on the introduction and application of the words for polygon and non-polygon. Students were also introduced or reintroduced to the words for: straight, curved, quadrilateral, open, closed. Many of these words have broader everyday connotations. The names of basic shapes (e.g., circle, rectangle, triangle) seemed to be familiar to students already. Using principles from conversation analysis, participants' orientations were used to language that they themselves saw as formal or informal.

POLYGON OR NON-POLYGON?

Prior to introducing the terms for polygon and non-polygon, the teacher took the students through two activities. First, she worked with the class on different ways of classifying the students in the room, including, for example, students wearing jeans vs. those not wearing jeans. The teacher referred to the resulting two groups as "les jeans" and "les non-jeans", with emphasis on "non". The discussion therefore introduces the students to the use of the prefix "non". Second, the teacher handed out packets of regular and irregular shapes to students and asked them to work in small groups to sort the shapes into two distinct sets. She invited different groups to show how they had separated their shapes and to explain how they had divided them. Students often struggled to explain their thinking in a way that the rest of the class and the teacher could make sense of, such as when they said (in French) "*I don't know how to say it in French*" or "*how do you call those*" or pointed, traced straight or curved lines with their hands or used words like "*this*" or "*like that*".

Next, the teacher introduced the terms "polygone" and "non-polygone" and drew examples of each on the blackboard. She asked the students to examine her drawings and deduce what a polygon is and what a non-polygon is. She said [1]:

> ok donc il y avait plusieurs façons de classer les figures […] une des façons (.) la plupart l'ont trouvée (.) il y a des figures qu'on appelle les (.) polygones (2) et les autres (.) qu'on appelle les (.) non (.) polygones […] ok c'était bon c'était très bon votre façon aussi on va en reparler plus tard (.) aujourd'hui on va plus voir les autres (.) donc polygone je vais te dessiner des exemples de polygone […] dans les non polygones il y a ça ça ça (6) avec mes

> dessins (.) et tu capable de m'expliquer qu'est-ce que ça veut dire (.) polygone (.) et qu'est-ce que ça veut dire non polygone? (.) explique-moi la différence (.)
>
> *ok so there are lots of ways to sort shapes […] one of these ways (.) most of you found it (.) there are shapes that we call (.) polygons (2) and the others (.) that we call (.) non (.) polygons […] ok your way was good it was very good as well we're going to come back to it (.) today we're going to look more at the others (.) so polygon I'm going to draw you some examples of polygons (…) for non polygons there's this this this (6) with my drawings (.) can you explain what it means polygon (.) and what does non polygon mean? (.) explain the difference for me (.)*

The teacher's introduction features several sets of differences: between the students' distinctions and the teacher's; between the two groups of shapes that the teacher draws on the blackboard; and between polygons and non-polygons. These differences are in dialogue with each other; making sense of the word and the concept "polygon" arises through the differences between the two groups of shapes on the blackboard, between the different ways of classifying shapes that preceded this moment, and so on. This approach captures a little of the tension between formal (unitary) and informal language (heteroglossia) to which Bakhtin refers. The teacher acknowledges the students' ways of classifying shapes, saying they were "very good", but sets them to one side in order to focus on the more formal terms of polygon and non-polygon. As such, she implicitly constructs the students' classifications and language they use to express them as less formal.

In the next few turns, the teacher elaborated on the meaning of polygon and non-polygon, with reference to the examples on the blackboard. She emphasised, in particular, the need for the sides to be straight and the shapes to be closed, pointing to examples on the blackboard as she spoke:

> les polygones c'est une ligne (.) droite (.) des lignes droites une ligne qu'on appelle brisée (.) ^ça veut dire quand elle est brisée comme ça c'est quand il y a plusieurs côtés^ et fermée (2.5) s'il y a les lignes courbes ou si la ligne elle n'est pas fermée (.) automatiquement c'est un non polygone (.)
>
> *polygons it's a straight line (.) straight lines a line that's called broken (.) ^that means when it's broken like that it's when there are several sides^ and closed (2.5) if there are curved lines or if the line isn't closed (.) automatically it's a non polygon (.)*

Again, the interaction between informal and formal language is apparent. For example, the teacher uses the formulation "une ligne brisée" (literally 'a broken line' but meaning rather something like a 'jointed' line). This formulation appears on the worksheet she gave out just after the above quotation. The formal definition on the worksheet reads "Un polygone est une ligne brisée fermée, tracée sur une surface plane." (*A polygon is a broken, closed line, drawn on a plane surface*). The teacher also explains 'brisée', using more informal language ("*when it's broken like that it's when there are several sides*"). Both formal and informal language are marked: the word "brisée" is preceded by "*that's called*" indicating a new term; the subsequent clarification opens with "*that means*", which signals a more informal formulation.

The worksheet also featured a set of shapes, labelled A-L. The students were asked to list each shape in a table, in columns labelled "POLYGONE" and

"NON-POLYGONE". After the students had worked on this task, the teacher went through the shapes with the whole class. For each shape, the exchange had a similar structure: the teacher nominated a student and stated which shape; the student stated if they thought it was a polygon or a non-polygon; the teacher asked why; the student provided a reason; the teacher revoiced or clarified with the student, in some cases leading to a new classification. For example:

Teacher	le *C* [E38] le *C* tu l'as mis dans quelle colonne?	C *[E38] which column did you put* C *in?*
E38	non polygone	*non polygon*
Teacher	non polygone (.) pourquoi?	*non polygon (.) why?*
E38	parce que c'est (.) pas ligne droit	*because it's not a straight line*
Teacher	ce n'est pas ligne droite c'est ligne (1.5) cou:rbe (.) garde je vais t'écrire ici (.) une ligne ça peut être droite (.) ou cou:rbe (2) k? bravo	*it's not a straight line it's line (1.5) look I'll write it here for you (.) a line can be straight (.) or curved (2) okay? well done*

The teacher's revoicings make small adjustments to the students' formulations. These adjustments are at different times to grammar, syntax, pronunciation, word choice or to mathematical distinctions. In the above exchange, E38 says "ligne droit" (the final t is silent). "Ligne", however, is feminine, so the standard adjectival form that follows would be "droite" (the t is sounded). The teacher's revoicing adjusts this and then elaborates, adding in "curved" as the contrast with straight. She then writes the two words on the blackboard. Over the course of 12 exchanges (for shapes A-L), some abbreviation occurred: students accounted for their classification without being prompted, for example. One student explained that her shape is polygon "*because the lines are straight and closed*" to which the teacher replied "*perfect*".

The lesson moved on to look at other attributes of geometric shapes. At the end of the period, however, the teacher asked a student to define polygon:

E53	un polygone c'est comme forme qu'il a des lignes droites (.) et il n'y a pas de trous	a polygon is like shape that it has straight lines (.) and there aren't any holes
Teacher	ok lignes droites fermées (.) si je te demande c'est quoi un non-polygone: [E54]	ok straight lines closed (.) if I ask you what's a non polygon [E54]
E54	non polygone c'est comme ah il y a comme il y a un trou dans le carré ou les lignes sont c-courbes	non polygon is like ah there's like there's a hole in the square or the lines are curved
Teacher	donc il y a une ligne courbe ou une ligne qui n'est pas fermée	so there's a curved line or a line that isn't closed

In these brief exchanges, two students give an account of their interpretation of the meaning of polygon and non-polygon. Again, the interaction between formal and informal language is apparent. Both students use the informal word "*hole*", for example, while the teacher revoices each time using the more formal word "closed".

DISCUSSION

In the lesson described above, the language of mathematics precedes the teacher and her students. They must grapple with it in an attempt to make it submit to their intentions. For the students, this struggle includes their encounter with the otherness of new words, new distinctions or new ways of using language. The teacher also encounters otherness, in the students' diversity, of accents, pronunciations, non-standard French and their informal expressions of mathematics. According to Wegerif (2008), this otherness "contains them within it". In the above lesson, the students must try to see things as the teacher does. When she asks "*polygon or non-polygon?*", they must use her formal terms and her distinction to respond. The question contains them; it reflects a centripetal force towards a particular way of seeing shapes and doing mathematics. By the same token, however, when the students reply, using a more informal language of holes and lines and gestures and pointing, the teacher must try to see things as they do. Their utterances, then, reflect a centrifugal force that contains the teacher. Throughout, there is a dialogue between the two. Through this dialogue, the students and the teacher come to use language in new ways: the language of both changes through the lesson in response to the utterances of the other.

A dialogic perspective on formal and informal language in mathematics classrooms highlights a relationship between formal and informal that is not uni-directional. Rather than steady progress from informal to formal, these students work at both. The teacher, too, must make skilful use of varying degrees of formality. Of course, students need to learn formal mathematical language as part of learning mathematics, but this does not mean that informal language disappears; nor is it simply a scaffold to reach more formal language. Both are necessary; they will always be in tension.

Note

1. Transcription: (.) or (2) for pauses, ^ ^ for whispering, […] for omitted parts. The translations are my own.

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