DESIGNING INNOVATIVE WORKSHEETS FOR IMPROVING READING COMPREHENSION OF GEOMETRY PROOF

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This study explored the effects of innovative worksheets of geometry proofs on students' reading comprehension of geometry proof. The worksheets treatment was designed with reading comprehension strategies. One mathematics teacher and two classes of ninth-grade students who were taught by her participated in this quasi-experimental classroom study. While the experimental group was instructed with the innovative worksheets in two forty-five-minute sessions, the control group was instructed with their regular materials about "inequality" during winter vacation courses. ANCOVA was conducted on posttest and delayed posttest with pretest as a covariate. Results showed that the score of the delayed posttest of the experimental group was significantly higher than the control group although there was no significant difference in posttest. The study supports the value of a mathematics proof curriculum with reading perspective following similar instructional strategies.

INTRODUCTION

For learning content, visualization and dynamic construction are suggested (Hanna, 2000). For learning to construct proofs, investigation of propositions or conjecturing is included to inspire the need for proof, and validating proofs is helpful for understanding proofs and further constructing a valid proof (Koedinger, 1998). For example, conjecturing activities try to engage students into finding some patterns or properties from several numerical examples, geometric figures or situational phenomena. These kinds of activities are based on the practice of mathematicians, and they require students to formulate their own proposition.

How to coordinate the divergent essence of conjecturing and the convergent essence of proving is still questionable. Transforming the verbal representation of dialogue into the literal and symbolic representation of proof is an obstacle which students must overcome for understanding the nature of proof (Sfard, 2000). Listening, speaking, and doing proofs are considered necessary activities for learning proofs, and reading should not be left out. Solow (2002) suggested useful strategies to read and do proofs to college students. How junior high school students learn and accommodate these strategies and convert the strategic knowledge into action are still critical issues of learning mathematical proofs. Especially, how well junior high school students comprehend mathematics proof is required for analysis as the quality of teaching proofs with the perspective of reading literacy is evaluated.

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Yang and Lin (2008) viewed reading comprehension of geometry proof (RCGP) as an object of study in addition to a processing component of producing, validating or interpreting arguments. They had conceptualized RCGP; that is, understanding proofs from the essential elements of knowing how a proof operates and why a proof is accepted besides knowing proof methods or ideas and what it proves. Six facets -- basic knowledge, logical status, integration or summarization, generality, application or extension, and appreciation or evaluation-- were formulated.

Reciprocal teaching method has been one of the most outstanding strategy instructions (Rosenshine & Meister, 1994). It is designed to improve reading comprehension by teaching cognitive strategies such as question generation, clarification, summarization, and prediction. Students attempt to gain meaning from text using theses strategies (Palincsar & Brown, 1984). Both reciprocal teaching only and explicit teaching before reciprocal teaching focus on the instruction in the cognitive strategies and students' practice of these strategies. It at least takes more than three lessons in most of the reciprocal teaching studies (Rosenshine & Meister, 1994). It is worth trying to use cognitive strategies to design worksheets instead of training students to use these strategies if improving students' RCGP with less teaching time is required, especially when the number of researches on students' reading strategies of comprehending geometry proofs is still few.

Based on the understanding of RCGP and transactional perspective on reading comprehension, we tried to design innovative worksheets for improving students' RCGP. The purpose of this study was to evaluate the effects of the innovative worksheets designed with reading strategies on RCGP for ninth-grade students who had learnt geometry proofs in school. For the experimental instruction, students were asked to answer questions individually, and then discuss in groups or present before the whole class. The performance of experimental students on RCGP was compared to that of students who did not receive any instruction of reading geometry proofs but took a test of RCGP where their mathematics teacher explained answers to them.

METHOD

Design of Worksheets

According to schema theory (Anderson & Pearson, 1984), we can derive meaning of texts based on our preexisting knowledge, and the better understanding can be realized while proper schemas can be triggered. Information must be processed in working memory before modified schemas are stored in long-term memory (Nassaji, 2002). On the other hand, instructional designs that increase germane cognitive load are beneficial to students' learning because students are guided to focus on cognitive processes that are necessary for accommodating schemas (Sweller, van Merrie¨nboer, & Paas, 1998). Thus, the innovative worksheets are set to trigger or to structure students' schemas, and to reveal and to acquire cognitive processes by peer and class discussion.

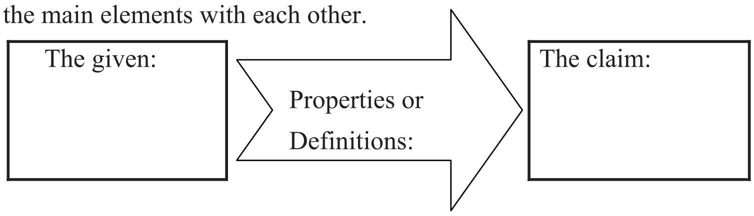
There are two worksheets. The worksheet ONE provided a proof and its corresponding propositions. The design of the following tasks were based on reading strategies of questioning, predicting, summarizing and clarifying (Brown & Palincsar, 1987; Palincsar & Brown, 1984) which are commonly used by good and active readers. The worksheet TWO provided another proof and its corresponding, and the following tasks are the same as the worksheet ONE. The ideas of using these strategies to design the worksheets are described in the following.

Questioning is to ask questions related to the text for monitoring and regulation comprehension of written materials. Students might have difficulties in creating their own questions while reading proofs. In this study we adopt this strategy to provide genetic questions, Q1-1:What is “the given” in the above proof?, and Q1-2:What is “the claim” in the above proof?, to prompt students to recognize the logical statuses of related statements and to monitor if they can identify them.

Predicting is to predict what will come next in the text you are reading. While reading a proposition and its proof, one should predict what can be inferred based on the given (forward, Q1-3:Thinking in the beginning of “the given” in the above proof, what can you infer in the next step?) or on the conclusion (backward, e.g. Q1-4:Which result can you derive from “$\overline{BC} = \overline{AD}$”directly in the above proof?). This might trigger relevant knowledge.

Asking for *clarification* if needed is one approach to modifying schemas. For examples, readers should point out what they do not understand or clarify whether their understanding is coherent. Q2-1:In the above proof, which properties are written unclearly in the given?, Q2-2:In the above proof, which proof steps do you not understand? Please circle them., and Q2-3:In the above proof, can you think which proof steps are redundant or unnecessary? Please delete them with straight lines in the above proof. are designed to check if students understand the meaning of this proof through self-evaluation after questioning and predicting guidance.

Summarization is to summarize several statements or paragraphs in simplified substance. For helping students understand a proof structure and synthesize the proposition and its proof steps, we provide a proof mapping as a visual display. Students are initiated to identify important proof steps and chain proof steps logically by attending to structure the critical elements of a proof through this proof mapping by Q3-1:Now, please use brief sentences to write down the main elements of the proof in the following mapping, and Q3-2: Please discuss in group. Write down the difference of the main elements with each other.



In this study, we also design tasks to ask students to think the predicting questions again and to compare their answers with the initial answers, to describe the underlying relationship of a proof mapping and to reflect their reading strategies based on their metacognition of what they understand and how they read by Q4-1:If you were a teacher, which problem do you formulate to let students write down the proof?, Q4-2: Do you have the same answer between the question you devise in (4-1) and (1-1) (1-2)? What is the correct answer?., Q4-3:Please go back to check (1-3) and (1-4), and you can revise the conjecture of (1-3) and (1-4) if you need., Q4-4: Referring to (3-1), please describe the relationship between the given, properties or definitions, and the claim., Q4-5:While applying the properties or definitions, what should you notice?, and Q4-6: While you read mathematical proofs hereafter, how will you read to easily comprehend proofs?. Theses questions are designed to reveal and acquire cognitive processes of reading geometry proofs.

Instructors and Instruction

The participating teacher had taught in junior high school for six years. She had taught ninth-graders' geometry proofs twice. She joined the first author to discuss the worksheets and the way to implement the worksheets. One of the two classes was randomly chosen to instruct with the worksheets. The other class was instructed with their regular materials about "inequality".

In the experimental group, students were asked to answer questions 1-1 to 1-4. The instructor asked one or two students to show their answer, and then discussed if the answers were plausible with all students; Questions 2-1 to 2-3 were adopted to check if students have sufficient pre-knowledge to understand this proof. The instructor can explain some properties, e.g. ASA, while students don't know it. Questions 3-1 to 3-2 provided a framework to summarize proof steps and asked students to discuss with peers. After peer discussion and presentation, the instructor explained how to summarize proof steps with the given, the applied properties and the conclusions. Questions 4-1 to 4-4 asked students to rethink what this proof proves and clarify their initial understanding and their summarization. To reflect their strategies of comprehending proofs, questions 4-5 to 4-6 asked students to point out the conditions of applying a property and to describe how to understand proofs.

In the control group, the mathematics teacher introduced the concept of linear inequality with one variable and the skills to find the solution of linear inequality. Students were asked to solve problems of inequality, discussed with peers and then explained to the whole classmates.

Subjects

One mathematics teacher and her two classes of 66 ninth-graders (14 to15 years old) participated in this quasi-experimental classroom study. These students who had leant formal geometry proofs in school were the subjects of this study, because their RCGP may not be advanced by the traditional geometry proof instruction (Lin & Yang, 2007).

One class of 9th graders (N=32) was instructed with the innovative worksheets while the other (N=34) was instructed with their regular textbook during winter vacation courses. However, only 24 and 26 students of the two classes respectively completed both posttest and delayed posttest. Based on the analysis of pretest, the absent students in the experimental group performed a little better than in the control group while compared to their class peers, and the absent students in the two groups performed a little poorer than their class peers.

Dependent Measures

Five quantitative measures were derived from this instrument of RCGP (Yang & Lin, 2008): basic knowledge comprehension, logical status comprehension, summarization comprehension, generality comprehension and application comprehension. The sum of the five measures represented the performance of RCGP. It took about thirty minutes to complete this test in both the two groups. Moreover, the instructors spent about fifteen minutes explaining the answers of this test for the students.

The posttest instrument as well as the delayed posttest for measuring RCGP were developed according to the operational definitions of the first five facets. The proof steps read in the posttest and delayed posttest were more than that in the pretest. The Cronbach's alpha reliability coefficients of the pretest and (delayed) posttest instruments were .84 and 0.73 for the ninth graders.

Procedure

This study consisted of four different phases: *Phase 1 Pretest.* Prior to the instruction, the questionnaire of Reading Comprehension of Geometry Proof (Yang & Lin, 2008) was administered to both experimental and control groups. *Phase 2 Intervention.* The experimental group is instructed with the worksheets in two forty-five-minute sessions. The control group was instructed with their regular materials about “inequality“ in two forty-five-minute sessions by the same mathematics teacher. The two sessions were part of winter vacation courses. *Phase 3 Posttest.* Two weeks later of the winter vacation courses, all students answered the posttest questionnaire. *Phase 4 Delayed Posttest.* About three months later of the winter vacation courses, all students answered the questionnaire which is the same as the posttest questionnaire.

RESULTS

The means and standard deviation on the total scores of reading comprehension of geometry proof (RCGP) from pretest, posttest and delayed posttest for the experimental and control groups were presented in Table 1.

	Experimental		Control	
Test (Full Score)	M	SD	M	SD
Pretest (29)	12.92	4.24	12.46	3.25
Posttest (29)	14.29	4.19	13.07	5.11

Delayed Posttest (29)	14.00	4.67	10.73	4.26

Table 1:Means and standard deviations on the total scores of RCGP for the experimental and control groups.

Regarding pretest, the mean scores for the control groups were a little lower than the mean score for the experimental group. However, the two independent samples t test revealed no significant difference among the two groups (t(48)=.428, p=.671). Analysis of pretest results provided further support that the groups are comparable. Students' performance on RCGP were really improved even if the raw scores on pretest, posttest or delayed posttest did not show trivial increase for the two groups, because the posttest and delayed posttest questionnaires were harder than the pretest questionnaire.

Regarding posttest and delayed posttest, the distributions of the scores on RCGP for each group followed a normal distribution, according to the Kolmogorov–Smirnov one-sample test. Before using the analysis of covariance, posttest scores were transformed with the power of square for fitting the assumption of equal variances of the scores between the two groups. Test of homogeneous regression coefficients also confirmed equal regression slopes between the two groups while either posttest or delayed posttest scores were dependent variables. ANCOVA was conducted on posttest and delayed posttest with pretest as a covariate.

There was no significant main effect on posttest for the two groups (F=.38, p=.541); however, there was a significant main effect on delayed posttest for the two groups (F=6.53, p=0.014). Results showed that the delayed posttest of the experimental group was significantly higher than the control group. The progress from pretest to delayed posttest in RCGP could be attributed to the treatments of innovative worksheets while the two groups were taught in the same school by the same mathematics teacher during the winter vacation and the following three months.

The low observed power of posttest (d=0.093) indicated that a larger sample size for testing the effect of the experimental instruction on RCGP is required. Although not significantly different, the changes following explaining a test of RCGP and teaching reading proofs were somewhat larger than the changes following explaining a test of RCGP and teaching inequality. Delayed posttest intended to identify whether the performance on RCGP changes were retained or lost, suggest that these changes were retained just in the experimental group (d=0.706). The control group showed significant decrease from posttest to delayed posttest.

CONCLUSION AND REFLECTION

Significant difference in the total scores of RCGP was observed between the two groups regarding delayed posttest with pretest controlled, though no significant difference in the total scores of RCGP was observed between the two groups regarding posttest with pretest controlled. This proved that after two forty-five-minute sessions of teaching, the significant effect can be observed after 12 weeks. Moreover, the retained progress in RCGP is related to the instruction of reading geometry proofs and not merely due to retesting conditions or spontaneous development.

There are some explanations for the absence of short-term effects. First, the test-retest condition might make statistically equal progress in the two groups for a short period of time, because reviewing a test is also a kind of learning. Thus, the delayed posttest is always thought to show the effectiveness of instruction. But, why cannot the progress of the control group last for a longer period of time to the delayed posttest? Based on schema theory (Norman, Genter, & Stevens, 1976), knowledge representation consists of nodes and networks, and meaningful learning is to interconnect the nodes by links. Our control group might have not made nodes or networks store in long-term memory because test-retest condition did not really facilitate semantic linkage. However, the instruction for the experimental group might benefit students' cognitive operations, e.g. chunking (Batting & Bellezza, 1979) or inference making (Pressley, 2000), which might further affect students' learning after instruction. Thus, tracking cognitive operations which could be effective over time is our next step for modifying the innovative worksheets and integrating different models of text comprehension.

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