GENERIC PROVING: UNPACKING THE MAIN IDEAS OF A PROOF

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Learning proofs is known to be hard for students of all ages. Generic proving (GP) is an interactive didactical process for addressing this difficulty. GP is situated in the context of exemplification in general, and generic examples in particular. When we want to convey vividly the essence of a mathematical concept, we use a *generic example* – an example complex enough to "see the general in the particular" (to quote Mason and Pimm). Thus, 36 is a good generic example of *perfect square*, but 4 is not (being too special). When we want to convey the essence of a theorem, e.g., *a perfect square has an odd number of factors*, we can still use a generic example: 36 has 9 factors (namely, 1, 2, 3, 4, 6, 9, 12, 18, 36). This example, however, does not give us a clue as to *why* the theorem is true. For this we use a *generic proof* – a proof carried out on a generic example. Thus, we list all the different ways 36 can be written as a product of two factors: $36 = 1 \cdot 36 = 2 \cdot 18 = 3 \cdot 12 = 4 \cdot 9 = 6 \cdot 6$, from which it is evident that the factors come in pairs plus one stand-alone, thus totalling an odd number (specifically, $9 = 2 \cdot 4 + 1$). In general, a generic proof is intended to convey vividly all the *main ideas* of the proof, separating out the creative parts from the technicalities of notation and formalism of the complete proof. We have now seen an example of a generic proof, from which some of its advantages could already be gleaned, but this example is too simple to be considered a *generic example of a generic proof*. Many subtle and important issues would only surface in the context of more complicated proofs.

In the first part of the Working Session, participants will analyse in small groups some richer examples of GP, where the main ideas of the proof emerge gradually in the context of one or more *partial generic proofs*. In the second part, we will use the examples of GP generated in the first part, to reflect from several perspectives (mathematical, educational and philosophical) on the following more general issues.

- What is a good generic example in the context of a generic proof?
- What are the strengths and weaknesses of a generic proof?
- How big is the gap between a generic proof and the complete proof of the same theorem, and how can it be bridged?
- Not all proofs are equally amenable to a generic version. Can we characterize the proofs (or parts thereof) that are so amenable?

2009. In Tzekaki, M., Kaldrimidou, M. & Sakonidis, H. (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 297-297. Thessaloniki, Greece: PME.