MENTAL FUNCTIONING OF INSTRUMENTS IN THE LEARNING OF GEOMETRICAL TRANSFORMATIONS

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A case study is presented on what is learned by a very advanced Math student in a ninth grade class (fifteen years old, approximately). This case was obtained from an exploratory study[1] *carried out in a classroom of eighteen students when the employment of certain cultural artifacts was introduced to approach the theme of basic geometrical transformations. The analysis of the activity from a communicational approach to learning (Sfard, 2001) was useful to detect objectification of the geometric properties involved. Also, probable use schema construction derived from the manipulation of Cabri-II*[2] *software and of large jointed machines allowed for hypotheses to be put forward on the realization of internalization processes (Mariotti, 2002).*

THEORETICAL FRAMEWORK

The first step for all socio-cultural and historic points of view on the studies of mind (Cole, 1995, p. 190), is the assumption that the defining characteristic of the human species is its need and ability to inhabit an environment transformed by the previous members of its species. Such transformations, together with the transfer mechanism for these transformations from one generation to another, are the result of an ability/proclivity among humans to create and employ artifacts. Artifacts, in turn, are features of the material world incorporated into human action as means of coordination or articulation with the physical and social environment.

Since school use of computers and new technologies is on the rise, it becomes urgent to identify key points around which to organize their use in fostering diverse educational processes (Mariotti, 2002, p. 697).

One of the current lines of research into processes of semiotic mediation, specifically cognitive processes of instrumental genesis, with the source of its analysis being the self-same nature and manipulation of the artifacts employed (Mariotti, 2002).

This approach (idem, p. 703) sees an instrument as the unity between an object (any artifact, such as a technical mechanism) and the organization of potential actions or plans

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[1] Details of this exploration may been found in the research report "   Coordinating mediation of activity in the learning of geometrical transformations," published in the proceedings PME-NA 2002.

[2] In particular, the geometric transformations' menu was used jointly with the Help command, which displays a legend that *explains* the general features of the transformation in question and *names* the elements therein. This Help command plays a central role in our study, as could be seen on the research report "Coordinating mediation of activity in the learning of geometrical transformations," published in the proceedings PME-NA 2002.

for use, which thus constitute a structured set of invariants corresponding to a class of possible operations. These schema function as organizers for the user's activities.

According to Vygotsky's notion of internalization (Mariotti, 2002, p. 706), the internalization process can transform tools into psychological tools when an internally oriented tool becomes a "psychological tool" and molds new meanings. It is in this sense that a tool can function as a semiotic mediator.

Mariotti (idem, p. 707) points out that a key facet of the internalization process is the distinction between the use of an artifact with an external or internal orientation together with a transition from the external to an internal orientation. In this view, an artifact and the schema for its use become an instrument, which can function internally, and once implemented contains the potential to shape new meanings.

In addition, Crawford's (1996, p. 137) review of the contributions by Russian theorists into the functioning of the mind reports Davidoff's position that the internalization process entails not only a transition from the external to the internal plane, but also a transition from collective to individual activity.

In the framework of this transition, this paper presents one student's results at the end of each practical activity. This high-achieving student is named Guillermo. Just as in Crawford's work (ibidem), the collective activity took place as a joint practical activity and was expressed through communication and language.

Finally, recent progress into a communicational approach to learning (Sfard 2001) in the classroom allows for understanding the reflection process as a mediator among the intellectual and personal components of cognitive activity. This is in accordance with Semenov (Crawford 1996, p. 139) having in mind the thought phenomena that occurred during problem solving.

The latter would be equivalent to saying (Crawford 1996, p. 137) that the subjects establish connections starting from reflecting on the objects of a problem and moving toward consideration and evaluation of their own acts and strategies throughout the process of cognitive activity.

AIMS, METHODOLOGY, AND RESULTS

The observations detailed below were collected from an exploratory study (see endnote (i)) carried out in a ninth grade class in a public school in Mexico. As may be seen by his actions, Guillermo is an outstanding student within the classroom observed. With ease and accomplishment he worked, paired with a student named Aníbal, all the tasks presented in a set up of then practical activities (50 minutes each) implemented in the classroom. Video logs were obtained of Guillermo's communication, especially because each pair was asked to describe what they had done at the end of each designed activity.

It might soon see that their descriptions reveal differing use schemes for the artifacts, as well as an internalization or assignment of new meanings. The latter probably will be made manifest through a later example of his solution to one of the problems assigned.

First, it is intended to present empirical references to the use schema construction or instrumentalization; then the probable articulation of this instrumentalization, together with the possible internalization and objetification the student achieved.

Instrumental genesis

At the end of the reflection and translation learning activities, during his description of what he had accomplished, Guillermo makes reference to the various moments of the activities performed during the two learning scenarios that were set up (see the bibliographical reference as in the endnote (i)). First, the student briefly mentions what he did within Cabri-II. Then, when describing his work with the pantographs, it might be seen how he advances toward characterization of geometry's invariants of the reflection and translation. The description might show how he mentally incorporated the use of other geometric tools (like the ruler and compass, which at no time were physically present) to that of the jointed machines instrumented here.

Guillermo describes activating Cabri-II's Reflection command as making the initial and image objects "move in opposite directions:"

Guillermo (hereinafter G): What we saw with this machine [the jointed machine for reflection or axial symmetry] was the same as for axial symmetry, what we were seeing, for example... If we took these by one point, then they are moved [two corresponding points in an initial triangle and its image] [it is noticed that he is manipulating the jointed machine, moving its drawing guide] they go opposite ways. Like [what] we were seeing within the Cabri software... What we were verifying here [with the jointed machine] was that axial symmetry is practically a reflection of the original figure, if we move [the design guide] to the left ... As an example, when we move the original figure from left to right, the reflection goes from left to right. [his partner Aníbal signals he has made a mistake] Yes. So what did you notice about the relationship?

Interviewer (hereinafter I): Let's see. Let's see. Once more.

G: If you move from left to right, [G now handles the jointed machine's design guide] this [the jointed machine's tracer] moves from right to left.

I: And so where will the axis of symmetry be here?

G: The axis of symmetry would be this line**.** [he points to the bar that serves as the machine's axis of symmetry]

I: And so if I say, put a point here, [I sets a point on one of the figures the students drew earlier, on the same work plane under the machine] how will I know which is the corresponding one — without using the machine?

G: This one… [the point the interviewer set down] ... Tracing a line [he picks up a ruler and places it over the jointed machine, perpendicular to the axis of symmetry] that is perpendicular. We make it go through the point and, in this figure over here, [in the image] we can calculate more or less where it will appear, which will be round about here.

I: Yea. But if I don't have any figure, [the interviewer places a point anywhere outside the figures the students have drawn] say, I just have this point, so, how do I know which is the corresponding one over there [in the image plane]?

G: It'll be just the same, doing a line that crosses perpendicularly.

I: Yea, so that it crosses, for example, here, [the interviewer places the ruler over the axis of the machine] I trace a perpendicular...

G: Right from this one, [the student shows the perpendicular's point of intersection, which was signaled physically with the ruler the instructor placed on the axis of symmetry] we could take a compass, [the student opens his hand, he's forming a compass with his fingers] and draw a circle that goes the radial distance from here to here, [he now shows with his fingers a perpendicular segment from the drawn point to the axis of symmetry] so here, [showing by hand a portion that would be the complement of the diameter of an imaginary circle he has traced with his finger compass] where the line intercepts the circle, [the imaginary circle] we'll get the point.

Comment: In reality, in the preceding passage Guillermo's statement on the reflection properties (or axial symmetry) has both degrees of specificity and generality that indicate he has abstracted and made objective the invariant property of geometrical reflection. It is also noteworthy that the mental experiment he is performing in response to the question about how we find the reflection of any point placed on the work plane, as well as the way he accompanied these actions ostensibly (by moving his hands). Further, it is interesting to note how this segment shows that from the interaction between the Interviewer (I) and the student (G), G makes progress in the precision of reflection's principal property: any two corresponding points under reflection will be at the same distance of the axis of this transformation.

Internalization

As explained above, Guillermo is an outstanding student, who even from the first description of what he performed on dilation with the software — unlike the executions of his classmates — achieved remarkable precision and signification on the use of terms and the degree of generality and specificity of his symbolic representations:

I: [as the students describe their computer work] Which … which longitude?

Aníbal: What we have between this… [indicates the computer screen] What there is between A and A', and between ...

G: [Points at the monitor] From A' to point O, or the dilation [center], from B' to the point, [at the center of dilation O, which he indicates with his finger]; from C' to the point, [idem previous note] and from A, B, and C to the point [idem previous note], so you can get them and divide them and see if they coincide with... with... What is it? With the...

A: With the scale?

G: Yea ... With the scale.

I: Or the dilation factor. Yea, scale. That's fine, or the dilation factor.

While the description Guillermo and Aníbal draw out is now quite coherent insofar as the properties of dilation, in fact, what is of interest to us right now is the description Guillermo composed for the written report at the end of the task.

Comment: Noteworthy here is the perfect description of the proportional relation based on the Thales configuration he obtained when tracing the straight lines through points O and A’, O and B’, O and C’ –that he has just mentioned—and taking any one point from ABC, an initial triangle he drew. As can be seen in Figure 1, he denotes this point (the elected arbitrarily, and the corresponding image) by using dots (···) and stars (*). He placed these dots and stars above the initial and image triangles, associating them with an arbitrary point he has elected to be over the initial or starting triangle.

The proportional relation, perfectly enunciated by Guillermo (between the triangles he sees based on the Thales configuration: A'/··/O and A/·/O; or between the triangles A'/**/O and A/*/O (see Fig. 1)) appears neatly written on his worksheet. As we can observe, Guillermo achieves perfect use of the terms that are the most conventional used for dilation.

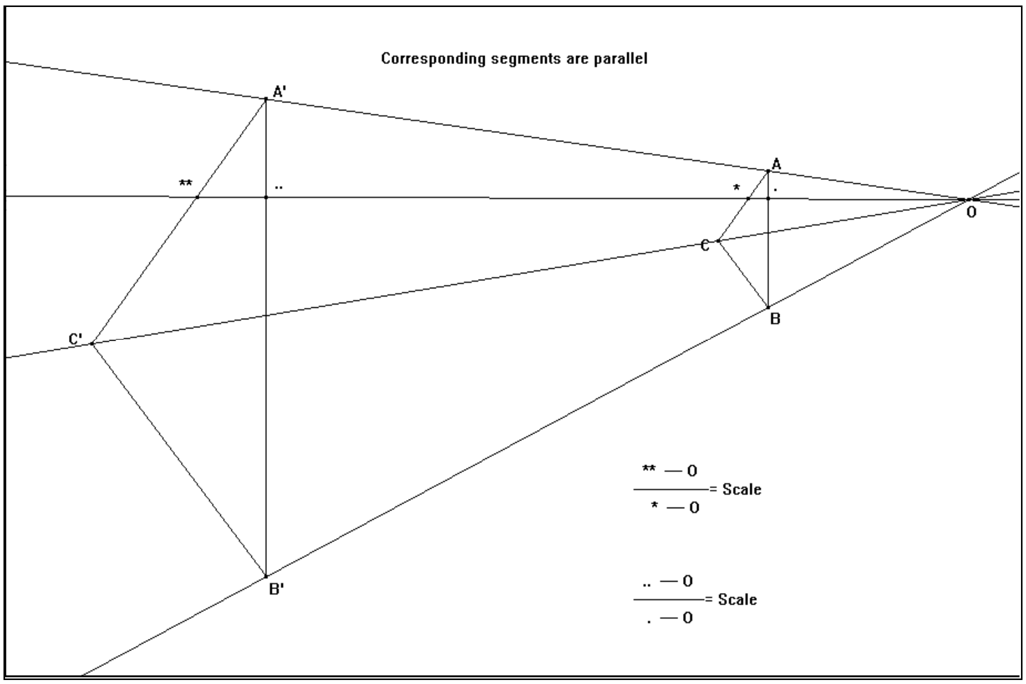

Figure 1. Guillermo’s description (identical, from his worksheet) on dilation after practicing within Cabri-II.

Notwithstanding the polished description of the geometric properties that the student observed after practicing with the software, it is still possible to measure improvement or progress in how precise the description of the transformation was after handling the Scheiner pantograph. The following it is a transcription of Guillermo written report:

- The transformation is a dilation on a scale of 2:1, that is, double
- The center of dilation is * where it is the base of the machine
- The Thales configuration is another characteristic feature of dilation
- The sides and corresponding points are parallel
- The angles are equal

Comment: In this direct quote there appears an explicit reference to the angles of the figures, which would complement his earlier description in answer to the computer worksheet on dilation.

Solving problems and idiosyncratic evaluation

To exemplify the signification and use of the geometric properties he learned, we cite Guillermo's answer to problem 2, which read like following.

Problem 2. "Points P' and Q' are the points reflected of P and Q in respect to line L.... Can you find line L using your ruler, but without measuring? Which lines can you trace to find the reflection axis?"

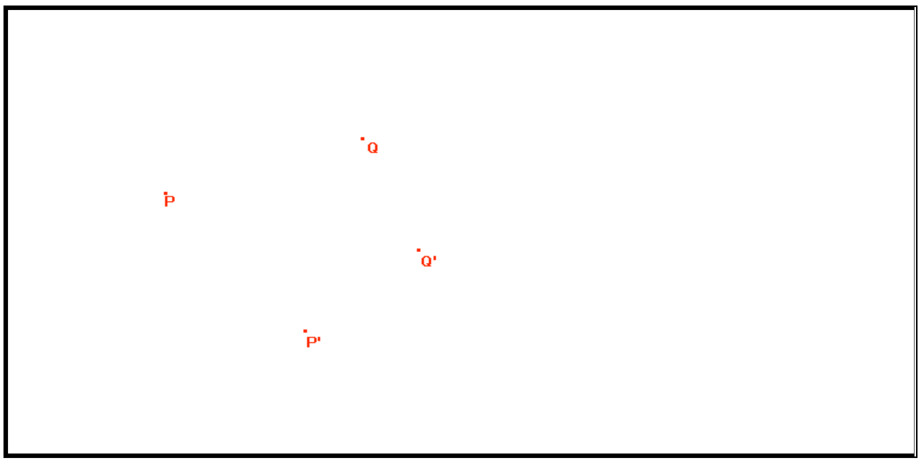

Figure 2. An image of Problem 2.

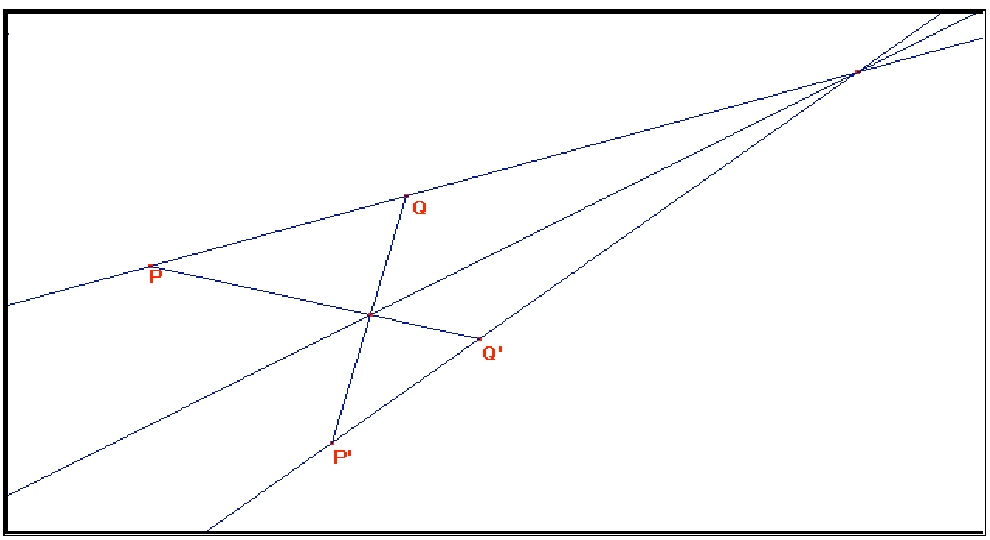

Figure 3. Guillermo's graphic solving procedure to Problem 2.

The description of Guillermo's solution was the following:

I: Let's see. Explain this [what you did] to me, Guillermo.

G: First, since the points are like this, in a trapezoid, I took point Q and point P', [see above for the figure and lines] and made a segment, just like with P and Q'. Then to get the other point, to make the straight line, I extended the segment that goes from P to Q and from P' to Q' and got a point of intersection. So this point of intersection and this one [points to the first point of intersection obtained, to the intersection of the diagonals of quadrilateral PQQ'P'] join with a straight line. And the straight line comes up, which is the one that makes the axis of symmetry.

The final passage also might concern the question of internalization and/or signification of the terms in use. We present a segment of the last interview with Guillermo, where he was asked to offer his opinion about the work he carried out in all the sessions:

I: Let's see Guillermo, tell me how you feel about these work sessions.

G: Yea, well, alright.

I: Right. Has what we've done been interesting for you?

G: Yea, because it made us think awful hard.

I: Yes. Did you like the computer work?

G: Yea, just that it's easier with the computer than doing it here, directly. [he points to the jointed machines]

I: But the work with these machines, does it seem productive? I mean, it leaves you with something. Interesting, isn't it?

G: You learn more than with computers, cause you do the work.

I: You learn more with computers, is that what you think?

G: No, you learn more here, [points to the jointed machines] cause here you do the work and the tracing and on the computer the only thing you do is lead it.

It is clear that Guillermo finds the work with the jointed machines more significant. From our point of view this is so due to the role these artifacts played here in order to reflect on the actions taken to solidify or objectify the notions whose learning was in play.

Students might naturally tend to assign greater value to learning gained once it an objectivization process has taken place, as they will contrast their current capacities against those they had previously been able to perform, during a previous stage. Upon completion of the activities, Guillermo feels armed with techniques and notions that provide problem-solving abilities (see for example his solving procedure to Problem 2), about the geometrical problems that he may not have been able to confront at the conclusion of the first stage of learning.

Idiosyncratic use of contextual terms (as Guillermo has just shown) to evaluate attainments fulfilled might be evidence of cognitive attainment that probably is only reached through accomplished internalization of instruments in use.

CONCLUSIONS

The actions performed in the second scenario* were structured inversely from those implemented in the first. In the first scenario, the support provided by the Cabri-II software allows initial activation of the invariant geometric properties involved in the chosen geometric transformation, by accessing the geometric transformations' menu. On the other hand, in the manipulation of pantographs, the necessary first step was to create a drawing and establish a comparison with the drawing obtained simultaneously from the tracer of the jointed machine in turn. Straight away, students were asked to determine what kind of geometric transformation was being used. It was in fact, an inversion of the procedure compared to the actions performed within Cabri-II.

In this way, complementary to what was obtained from using the software, an objectification was observed in the second scenario, resulting from manipulating of the jointed machines, or pantographs. It evidenced greater comprehension of the notions in relation to a more appropriate mathematical use of the terms. The evidence that students have established numerical or specific relationships between the traced figures might demonstrate how the terms or symbols in use could become adequate representations of invariant properties of geometric transformations.

Finally, it might be that the probable internalization that may have occurred throughout the implementation of learning scenarios set up here — speaks of a research productive issue into mediated action (Wertsch, 1993), which could be developed at the classroom, as this piece of research attempted to show.

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