THEMATIC WORKING GROUP 4
ARGUMENTATION AND PROOF

Maria Alessandra Mariotti
Univeristà di Siena
Kirsti Hemmi
Stockholm University
Viviane Durand Guerrier
Université de Lyon

The contributions collected in this section differently address the issue of proof and argumentation, offering a quite varied spectrum of perspectives, from both the point of view of theoretical frameworks assumed and that of issues in focus.
The richness of contributions' diversity gave the participants the opportunity of a fruitful discussion far beyond the need of sharing a common terminology, while the reflection, carried out at the beginning of our working activity highlighted the problematic relationship between argumentation and mathematical proof from the diversity of our theoretical and cultural backgrounds. The papers presented and discussed during the working sessions at CERME5 are collected in this chapter, organized according two different main themes.

1. Models and theoretical constructs to investigate argumentation and proof

The first group of papers exemplifies how different theoretical constructs may contribute to shape investigations, directing the researcher both in selecting the questions to be addressed and the ways to look for possible answers.
The role of epistemological analysis emerges in the paper of Castagnola & Tortora, where the famous Euclid’s theorem on the infinity of prime numbers is presented as a paradigmatic example to discuss students' difficulties and to propose possible means for their overcome. The epistemological issue is also clearly addressed in the paper of Deloustal-Jorrand, where mathematical implication is analysed from three different points of view: formal logic point of view, deductive reasoning point of view, sets point of view. A *didactical engineering*, based on the assumption of the necessity of make these points of view interact is carefully described and its implementation discussed, showing how a suitable situation can raise the issue of implication. A complementary example is presented in Gibel's paper, where, in the frame of the *Theory of Didactic Situations (TDS)*, the author discusses the inadequacy of the situation that fails to engage students in solving a problem and consequently does not make it possible for the teacher to bring the students to valid mathematical reasoning.
Besides approaches consistent with general theoretical models, such as that of the TDS, specific theoretical constructs were presented, elaborated for the particular aim of analysing issues related to argumentation and proof. This is the case of the papers presented by Pedemonte and by Antonini & Mariotti. The construct of Cognitive Unity combined with the Toulmin model provides Pedemonte a powerful tool to

analyse and discuss the complex relationship between argumentation and proof in the special case of producing a conjecture. Antonini & Mariotti discuss the complexity of indirect proof using an interpretative model set up as refinement of the notion of Theorem introduced by Mariotti (200?).
Styliadis & Styliadis bring to the attention of mathematics education researchers a rich body of psychological research on deductive reasoning, related to the well known paradigm of mental models (Johnson-Laird, P.N., 1983). Such a theoretical construct may offer new insight not only in identifying important issues that require research attention, but also suggesting an interdisciplinary and collaborative approach to the problem of promoting proof in students' learning of mathematics.

Still a different perspective and a new theoretical model is drawn from cognitive psychology (Kahnman, 2002) and chosen by Buchbinder & Zaslavsky. Their objective is that of describing the students' behaviour when they are asked to decide truth-value of a statement; the model of the dual process theory is applied in order to describe (and explain) the differences between modes of justifying the truth-value of a statement, according to the notion of "confidence" (respectively "lack of confidence") that the subject has in its truth or its falsity.
A theoretical model is set up purposefully by Timmermann to describe the relationship that an individual (either the teacher or the student) can establish with a proof. Distinguishing between structure, components and details textbook proofs are analysed so as their presentation in class. A possible failure in communication between the teacher and the students may explain difficulties, and can be described in terms of discrepancies between what each interlocutor has in focus: a component or a detail.
As is the case for investigations based on the development of a teaching experiment, a twofold objective characterizes the study presented by Fiallo & Gutierrez: on the one hand to design a didactic intervention, on the other hand to study students' performances. An unusual mathematical domain is selected, trigonometry, and a teaching sequence is set up with the intention of introducing students to proof; in order to show its potentialities a specific model is elaborated for the analysis students' learning achievements.
A more explorative approach is taken by Aylon & Even. Differently from the previous studies, characterized by being structured and directed by the theoretical frame selected, this study aims to examine and classify opinions about the role played by learning mathematics in the development of general deductive reasoning. The interviewees are persons involved in mathematics education and logic. In spite of an expected variability, the findings of this study show a certain convergence on considering the development of general deductive reasoning as a goal of mathematics instruction, and on the assumption that to some degree this goal is attainable.

2. Teachers' beliefs and teacher practice

The realty of the classroom, as emerged from some of the previous contributions, brings to the forefront the centrality of the teacher's point of view, both in terms of

beliefs and in terms of the practices that in such beliefs find their origins.
This perspective is explicitly taken in the contribution presented by Sergis, illustrating how teacher's view of what constitutes a proof and its functions influences the choice of what is to be integrated into one's own teaching practices. He proposed an interesting issue concerning the relationship between teachers' beliefs and practices and the presentation showed the high complexity of treating this issue and the need of elaborating specific methodologies.
Shifting the attention on teachers' practices and aiming to explain how successfully a teacher teaches proof, Ding and Jones propose to analyse teaching of geometrical proof, as it is developed in Shangai classrooms. The analysis is carried out by using the van Hiele model and indicates that though the second and third van Hiele teaching phases could be identified in the Chinese lessons, the instructional complexity does not allow the full sequence of phases to be recognized.
If a global description of a successful practice seems far from being achieved, a more focused contribution comes from Gibel's paper. The author highlights the crucial role of the situation for a student to engage him/herself in an argumentative reasoning, nevertheless the episode discussed in the presentation raised the issue of the role of the teacher in supporting the student in overcoming the difficulty.
Teachers' beliefs and teachers' practices become a particular issue when the mathematics class involves prospective teachers. A long term teaching experiment is presented in the paper of Camargo, Samper & Perry, aimed to introduce future math teachers to deductive geometry. Among a number of different aspects characterizing this teaching experiment, the authors discuss on the mediating role that a Dynamic Geometry Environment, Cabri, can play.
The case of trainee teachers is still in focus in the paper presented by Cusi & Malara. Beyond the general difficulties related to proof, the analysis of proof production in elementary number theory show the specificity of this mathematical context.
As far as the task is assumed to require the coordination of verbal and algebraic registers, the authors show how the difficulties encountered in the solution processes can be explained by the lack of coordination.

3. Final conclusions

After the short presentations of the papers and the following debate, the working group participants broke into small groups. The discussion in the groups was guided by a set of issues related to the theoretical and methodological elements raised by the debate.

1. Using formal models for investigating proof and comparison between different models for investigating proof.
2. Proof in the classroom: focus on the task, focus on the mathematical domains focus on the teacher.
3. Teaching experiments for investigating proof: methodological issues related to investigating proof in the school context.

It is difficult to report conclusions for a group that spend considerable time facing the complexity of diversity. What emerged, rather than points of unanimous agreement, is the convergence on the centrality of some issues.

In our discussions it became clear that there is a need to specify the meaning of the terminology we are using, in particular in relation to the aim of dealing with analogies and differences between argumentation and proof; but equally importantly there is a need to understand better their inter-relations and relevance in the context of mathematics and of mathematics education as well.

The variety and the richness of the different theoretical frameworks utilized in the different contributions stimulated not only the need of comparison, but also the curiosity of undertaking a possible integration and this, in our view, constitutes the main result of our working sessions in the spirit of CERME.