SWEDISH UNIVERSITY ENTRANTS' EXPERIENCES ABOUT AND ATTITUDES TOWARDS PROOFS AND PROVING

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Abstract

About 100 university entrants responded to questions about proofs and proving. The part of the questionnaire, which deals with students' attitudes, experiences and feelings, as well as their actual proving abilities, was analysed. Almost 50 percent of the students had had teachers who often proved statements but only about 30 percent had had the possibility to exercise proving both orally and in writing. The results show an interesting correlation between the students' experiences and their proving abilities. Students' attitudes to proofs and the learning of proofs were very positive and in some aspects these results differ from earlier studies (e.g. Almeida, 2000). However, caution must be exercised when interpreting these figures. Firstly, the questions were of dichotomous nature. Secondly, we must expect the students to be influenced by the mathematics setting in which the questions were posed. In future analysis of data I plan to further the investigation of these results. The present survey is a part of a deeper study to be developed concerning students' culture of proof in school and in basic university courses in Sweden.

1. Introduction

The role of proof in the Swedish schools has presumably diminished since the Swedish curriculum reform in 1969 (Skolöverstyrelsen, 1969), a development, which is similar to that of many other countries (e.g. Hanna, 1995; Niss, 2001; Waring 2001).

In the seventies, university teachers in Sweden noticed that many students in the basic courses were not capable to handle tasks, which dealt with proofs and mathematical theories. Teachers solved the problem by moving a part of such tasks to the upper courses (Boman, 1979). There still exists a gap between mathematics as it is taught in upper secondary school and university mathematics. A lot of students have difficulties in passing the examinations in the basic university courses.

Proofs are essential part of mathematics and that is why you cannot ignore such activities in mathematics education (Hanna, 1995). Students need a lot of practice in order to develop their understanding of proofs and their abilities to prove statements.

The aim of this study is to investigate what kind of experiences, feelings and attitudes students have concerning proofs and proving when they start to study mathematics. The aim is also to analyse the impact of the teaching the students received in upper secondary school on their proving abilities and feelings.

2. Research questions

The curriculum changes were reactions against the dominating role of proof and theory and the way proofs were previously taught, which did not make room for discussions (Niss, 2001). But what is the situation in Swedish mathematics classroom of today? In this paper the focus is on the following questions.

- What are students' attitudes towards proofs and the learning of proof when they start to study mathematics at the university?
- What are students' experiences of the treatment of proof in upper secondary school?
- What capacity do students have to prove some elementary statements?
- Is there some correlation between students' experiences and their proving abilities?
- Is there some correlation between students' experiences and their feelings?

One of our conjectures was that if students have only seen their teachers prove statements they might be anxious to investigate proving tasks in different ways. But we did not expect many teachers to often prove statements to the pupils.

3. Methods

A questionnaire consisting of two open questions, three multiple-choice questions, 28 dichotomous statements and two proving tasks was designed and distributed to the students at the beginning of their first term at the University of Stockholm in August 2002. Hundred students responded to the questionnaire.

Questions from some earlier studies were used in the present study. Almeida (2000) used a questionnaire consisting of 16 statements for each of which the students had to select one of five responses: strongly agree, agree, no opinion, disagree, and strongly disagree. He also gave an 'ideal' response to each of the statements, i.e. the response a professional mathematician might have given. Eleven of these statements were applied to the dichotomous part of the questionnaire. In this paper only two of them were analysed, namely "I can't see the point of doing proofs: all the results in the course have already been proved by famous mathematicians." ”If a result in mathematics is obviously true then there is no point in proving it."

Besides the proving tasks only eight of the dichotomous statements and one of the multiple-choice questions, namely those concerning attitudes, feelings and experiences, were analysed in this paper. In the first part of the analysis all the students with foreign examinations were excluded.

The proving tasks used in the present study have previously been used by Recio and Godino (2001) and were adapted to be used in this study. The students' answers to proving tasks were classified into five categories.

1. The student gives no answer or the answer is very deficient.

2. The student checks the proposition with examples or draws a geometrical picture.
3. The student justifies the validity of the proposition by using informal reasoning (without algebraic symbols) sometimes combined with examples and visual representations.
4. The student uses algebraic symbols, but fails in operating with them.
5. The student gives a correct (or almost correct) proof, which includes an appropriate symbolisation.

Almost correct proof means that the student has made some minor mistakes with signs. The categories differ in some aspects from those of Recio and Godino. Category 2 consists of their categories 2 and 3. Category 4 of R&G is separated to categories 3 and 4 in this study. This is because very few Swedish students had checked the propositions with examples.

After piloting the questionnaire we had to change the geometry question somewhat and insert a picture of adjacent angles with algebraic symbols. Very few students grasped the question without such a picture.

In order to analyse the impact of the teaching students got in upper secondary school on their proving abilities the responses were dealt into four groups depending on how the students responded to the following statements: *"I have had a possibility to exercise proving both orally and in writing in school."* and *"My teacher in upper secondary school often used to prove statements to us."*. Group 1 consists of the students who disagreed with both of the statements. Group 2 consists of the students who disagreed with the first statement and agreed with the other statement. Group 3 consists of the students who agreed with the first statement and disagreed with the other one. Finally Group 4 consists of the students who agreed with both of the statements.
In order to analyse the impact the students' experiences had on their feelings the students responses were classified into three categories. The negative responses consist of alternatives *b) nervous, d) dull, e) insecure* and some of their own descriptions, like "anxious". The positive responses consist of the alternatives *a) curious, c) eager* and some of their own descriptions, like "This will be easy". The mixed group consists of those who had chosen both kinds alternatives.
In this part of the analysis the students with experiences of mathematics after upper secondary school were excluded and the students with foreign examination were included.

4. A review of relevant research and theory

The role of proof in mathematics education has been studied in several countries during the last twenty years. (e.g. Hanna, 2000; Niss, 2001).

In Sweden such studies are largely lacking. Tomas Bergqvist's doctoral thesis To Explore and Verify in Mathematics (2001) is an exception. In the thesis he deals with the issue of how some pupils at the Swedish upper secondary school verify their solutions. In his analysis Bergqvist uses Balacheff's levels of proving. One observation he makes is that the students both wanted and were capable in using 'higher level reasoning' when verifying their solutions.

Almeida (2000) made a quantitative study of UK mathematics undergraduates and described their declared perceptions of proof. He also studied a sub sample of these students and analysed their actual proof perceptions and proof practices and compared these with their declared perceptions. He found some differences between them. In the present study the focus is on students' attitudes towards proofs and the learning of proofs.

In number of studies researchers have set out to classify pupils' levels of reasoning and proofs (e.g. Balacheff, 1988; Harel & Sowder, 1998). Some researchers, however, question the existence of a universal hierarchy of students' ability of proving (e.g. Hoyles, 1997). Hoyles (1997) argues that hierarchies of this kind (e.g. concrete/abstract or formal/informal) are largely artefacts of methodology. There are huge variations between different countries concerning teaching of proof. It is important to study feelings, teaching, school and home environment in order to find other than purely cognitive reasons to why students' responses may differ (Hoyles, 1997). It is also important to "ensure that the goals for including proof in the curriculum and how these are operationalised are clarified and taken account." (Hoyles, 1997, p. 7) The present study focuses on students' feelings when faced with a proving task and relates both the feelings and their abilities of constructing proofs to the teaching they have got in upper secondary school.

Recio and Godino (2001) studied university entrants' mathematical proof schemes and related these schemes to the meaning of mathematical proof in different institutional contexts. One of their main conclusions was that it was difficult for the students to produce deductive mathematical proofs. In this study the students' proof schemes are related to the students' experiences of the treatment of proof in upper secondary school.

5. Proof in the Swedish curriculum

The Swedish curriculum does not state very clearly the aims of introducing the students to proofs and proving activities. Only the main goals are stated. Local schools and teachers have the possibility of applying these goals in their own way. "The school in its teaching of mathematics should aim to ensure that pupils develop their ability to follow and reason mathematically, as well as present their thought orally and in writing." (Skolverket, 2002, p.60) One of the Criteria for 'Pass' (lowest mark of a three-level grading scale: Pass, Pass with distinction, Pass with special distinction) for any of the five courses A-E is that "pupils differentiate between guesses and assumptions from given facts, as well as deductions and proof" (p. 60-66). Furthermore one of the 'Criteria for Pass with special distinction' is that "pupils participate in mathematical discussions and provide mathematical proof, both orally and in writing" (p. 60-66).

6. Analysis of the results

6.1. Students' attitudes

The results from the present study show that the students have positive attitudes towards proofs and the learning of proofs. If we compare the results with the results obtained by Almeida (2000) we find that the Swedish students tend to view proofs in a more positive way. The most negative responses (about 2.5 mean with ideal response 5) in Almeida's survey concerned the two following statements. "I can't see the point of doing proofs: all the results in the course have already been proved by famous mathematicians." "If a result in mathematics is obviously true then there is no point in proving it."

In comparison, only 11 percent and 14 percent respectively of the Swedish students answered yes to these statements. Furthermore, as many as 90 percent of the students agreed with the statement "Proofs help me to understand mathematical connections." Most of the students wanted to learn more about proofs and would have liked to learn more about proofs in upper secondary school. These results can be interpreted in a way that supports the results of Bergqvist, namely that pupils in upper secondary school want to use 'higher level reasoning'.

But caution must be exercised when interpreting these results. Firstly, the questions were of dichotomous nature. Secondly, the questionnaire was distributed to the students at a time when they were about to enter their studies at the mathematics department. We would expect them to be rather positive towards a subject, which they had freely chosen to study.

6.2. Students' experiences and solving of the proving tasks

From the responses concerning the students' earlier experiences we notice that 29 percent agreed with the statement: "I have had opportunity to practice proving both orally and in writing in school." The results also indicate that contrary to our expectations there are still many teachers who often prove statements to pupils. Of the 87 students who participated in the study 48 percent agreed with the statement: "My teacher in upper secondary school often used to prove statements to us." Finally, 59 percent agreed with the statement: "I have met different kinds of proofs in school."

If we compare the results of the proving tasks with the results from the study of Recio and Godino we find that using particular examples as explanatory arguments was not as popular among the Swedish students as among the Spanish undergraduates. When proving the geometry statement only three of the Swedish students used particular examples or visual representations compared to 37.3 percent of the Spanish students. One explanation to this result can be that the Swedish students were conscious of the fact that some examples were not valid as general arguments but unconscious of the means by which to prove the statements correctly. Another reason may be that the students lacked the practical acquaintance of conjecturing and verifying statements in school. However, further investigations are needed to clarify these results.

We can observe that the percentage of students giving correct (or almost correct) mathematical proofs were less than 40 percent for both problems. Only 17.2 percent (32,9 percent in Spain) gave correct answers to the two problems. Many of the students, who used algebraic symbols, failed to operate with them.

TABLE 1

Frequencies and percentages of types of answers

Category	Arithmetic problem		Geometry problem	
	Frequency	%	Frequency	%
1	39	44.8	50	57.5
2	10	11.5	3	3.5
3	8	9.2	2	2.3
4	7	8.0	1	1.1
5	23	26.4	31	35.6

6.3. Some correlations

A clear tendency is visible in these results (TABLE 2-3). The students who stated that they lacked the experience of both own investigations and teachers' lectures of proofs in upper secondary school had very weak result on the proving tasks. Experiences of following the teacher's proofs gave some improvement to these results. Even though the possibility of practicing proving both orally and in writing had more a positive impact than following teachers' proofs alone the group with both of those experiences got the best results.
Our conjecture about students lacking own investigations of proving tasks cannot be validated since the sizes of the middle groups (2,3) are rather small.

Considering students' feelings when facing with proving tasks we can also see a tendency, although very slight (TABLE 4). Different kinds of experiences of proofs in upper secondary school seem to have a positive impact on the students' feelings.

TABLE 2

Solving of the arithmetic problem in different groups

Category	Group 1 31		Group 2 23		Group 3 9		Group 4 23	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
1	20	65	11	48	2	22	8	35
2	3	10	2	9	1	11	3	12
3	3	10	4	17	1	11	1	4
4	-	0.0	2	9	2	22	1	4
5	5	16	4	17	3	33	11	48

TABLE 3

Solving of the geometry problem in different groups

Category	Group 1 31		Group 2 23		Group 3 9		Group 4 23	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
1	22	71	15	65	3	33	8	30
2	-	0	2	9	-	0	1	4
3	1	3	1	4	-	0	-	0
4	1	3	-	0	-	0	-	0
5	7	23	5	22	6	67	15	65

TABLE 4

Students' feelings towards proving in different groups

	Group 1 31		Group 2 23		Group 3 9		Group 4 23	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Negative	14	45	10	43	3	33	7	30
Mixed	7	23	3	13	1	11	2	9
Positive	10	32	10	43	5	55	14	61

Discussion

What do these results tell us? According to the students many teachers in upper secondary school often prove statements to their pupils. This is not an account to be taken negatively. Pupils may learn a lot depending on how well the teacher explains the steps to be taken in proof making and on how explanatory the proofs are. As a teacher you can focus on different structures of proofs when showing the pupils how to do. Teachers' lectures of proofs also seem to have a slightly positive impact on students' abilities to prove statements.

However, our analysis indicates that in order to get the best result pupils also need to exercise proving both orally and in writing. This goal is also stated in the national curriculum of Sweden.

There is certainly cause for concern for the students in Group 1. It has to be pointed out that the students, who begin to study mathematics at university, often come from science programme, where the focus of education is on advanced mathematics, chemistry and biology. Group 1 could even become bigger if students also from other programmes answered the questions.

This survey indicates that there are both quantitative and qualitative variations in the ways the proofs are dealt with in upper secondary school but it is important to keep in mind that students may have different views and interpretations of the classroom activities than their teachers. In order to get a more varied picture of the actual teaching practices it is important to study local syllabuses, to interview teachers about their ways of introducing students to proofs and to analyse textbooks in search of different ways to present proof-making to students in upper secondary school.

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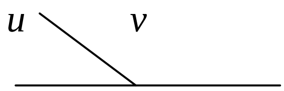
Appendice: The part of the questionnaire which is analysed in this paper

1. Proving tasks: Prove the following statements. A) The difference between the squares of every two consecutive natural number is always an odd number.

   B) The bisectors of any two adjacent angles form a right (90°) angle. (The angle bisector is the ray that splits the angle into two equal parts. *u* and *v* in the picture are adjacent angles.)



2. The question of students' feelings:

   When I get a task starting "Show that…" I most often feel

   a) Curious
   b) Nervous
   c) Eager
   d) Dull
   e) Insecure
   f) Some other way___________________________
   g) I have never got a task like that.

3. Write yes or no after the utterances depending on if you agree or not.
    a) I see no meaning with proving; famous mathematicians have already proved all the results. _______
    b) I have had a possibility to practise proving both orally and in writing in school. _______
    c) I have met different kinds of proofs in school. _______
    d) If a result in mathematics is obviously true there is no point of proving it. _______
    e) I would like to have learned more about proofs in school. _______
    f) My teacher in upper secondary school often used to prove statements to us. _______
    g) Proofs help me to understand mathematical connections. _______
    h) I would like to learn more about mathematical proof. _______